Traffic Speed Variance Modeling with Application in Travel Time Variability Estimation

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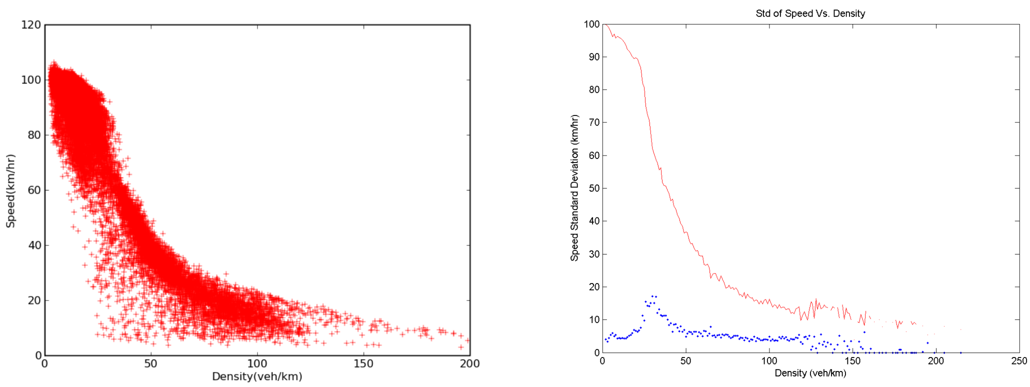
Abstract

Traffic speed variance is defined as a measure of the dispersion of space mean speeds among drivers. Empirical speed-density observations exhibit a structured traffic speed variance which has been found to be associated to the road accident rate, the fatality rate, and travel time variability. The objective of this paper is to propose a generalized traffic speed variance function to describe this structured variance and identify its potential applications. In nature, the proposed speed variance function is a response of the speed-density curve with two additional parameters. A series of logistic speed-density curve with varying parameters is used in the proposed traffic speed variance function with different performances. This traffic speed variance model will help to unveil the underlying mechanism of some empirical traffic features such as spontaneous congestion or capacity drop.

1 Introduction

1.1 Background

Nonconstant variability appears in numerous fields of scientific inquiry such as chemical and bioassay; traffic flow is no exceptions [1]. The wide-scattering effects in the equilibrium speed-density relationship has been popularly recognized by transportation researchers and professionals for many years (Referring to Figure 1(a)). The mean curve of the wide-scattering equilibrium speed-density relationship (Referring to Figure 1(b)) provoked sufficient modeling efforts using both deterministic and stochastic modeling techniques [2] [3]. However, the traffic speed variance (or equivalently the speed variability) which has been found to be associated with road accident frequency, travel time and its variability on both highway and major arterial roads [4] is not sufficiently addressed in literature. Lave [5] noted in his paper "Speeding, Coordination, and the 55 MPH Limit" that traffic speed variance kills, not speed. Based on his state cross-section data analysis of 1981 and 1982, he found that there is hardly any statistically discernable relationship between fatality rate and average speed, while there is a strong relationship to traffic speed variance. A variety of contributing factors (i.e., driver's lane changing behavior, number of lanes, vehicle heterogeneity [4]) affect the traffic speed variance, but the most significant one was identified by Garber as the difference between the design speed and the posted speed limit [6]. Instead of exploring the mathematical relationship between speed variance and crash rates, this paper focuses on the modeling, analysis, and the implications of traffic speed variation regarding the variance curve's nonlinearity and heteroscedasticity. The impetus to model speed variation arose from the need to empirically account for the observed traffic dynamics such as wide-scattering plots of the empirical fundamental diagram, and the onset of congestion as traffic densities vary. From a safety perspective, traffic speed variance directly relates to crash frequency and could help contribute to the identification of crash-prone locations on highway and arterial [4] [6] [7] [8].



(a) Speed-Density Scattered Plot (b) Speed Mean and Variance

FIGURE 1 The scattered plot of an empirical speed-density relationship and its mean, variance curves

A qualitative and quantitative description of traffic speed variance is significant to both researchers and professionals. Past research efforts related to this topic approached this problem with the following observations: (1) From a policy and safety view, speed variance and speed variance reduction are serious considerations for the setting of different speed limits for passenger cars, and heavy trucks [9], and highway work zone safety control [10]. Additionally, Graves et al [11] proposed a model of the optimal speed limit which explicitly recognized the roles of average speed, speed variance, and the enforcement level. (2) From a traffic operation and management standpoint, Saifallah [12] showed that the density of maximum throughput is near the density of maximum speed variance which agrees with the observation that maximum speed variance occurs at the critical density where capacity is usually obtained. Rakha [13] proposed a relationship between time mean speed and space mean speed variances, as well as space mean speed and travel time variance. Most recently, an on-going research project "Guideline Development for Minimizing Operating Speed Variance of Multilane Highways by Controlling Access Design" sponsored by the university transportation program is being conducted by the University of South Florida [14]. One of the project objectives is to quantify the influence of specific access design factors on speed variance using statistical techniques for the purpose if improving safety performance. This brief review of what has been done and what is ongoing is not intended to be complete but to emphasize that a better understanding of traffic speed variance is extremely important to the transportation industry.

1.2 Paper organization

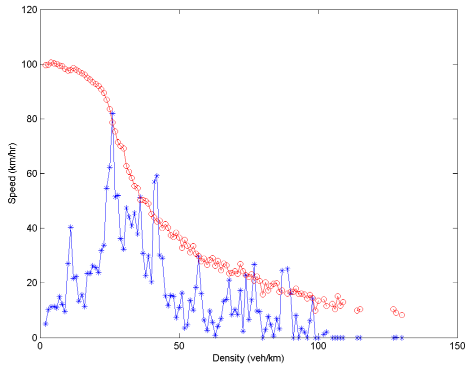
The remainder of this paper is organized as follows. In Section 2, a novel modeling of the traffic speed variance is presented with a focus on the nonlinearity and heterogeneity of the empirically observed traffic speed variance. In this context, a generalized traffic speed variance function is proposed in Section 2.2 in which the speed variance is a response of the speed-density curve with two additional parameters based on the smoothed shape of the speed variance curve (ignoring the local kinks).. The choice of the speed-density curves is provided in Section 2.3 in which a series of speed-density curves with varying model parameters are presented. In Section 3, the calibration of model parameters of the traffic speed variance function and the speed-density function are provided. An application example is designed to demonstrate how the proposed traffic speed variance model can be applied to estimate travel time and analyze travel time variability in Section 4. In Section 5, we briefly summarize our findings.

2 Novel Modeling of Traffic Speed Variance

From the structured empirical traffic speed variances (Referring to Figure 2), the traffic speed variance is modeled as a response of the speed-density curve with some additional parameters. So, we start the formulation of traffic speed variance function with its nonlinearity and heterogeneity.

2.1 The nonlinearity and heterogeneity of speed variance

In order to model the nonlinearity and heterogeneity exhibited in the structured traffic speed variance plots as can be seen from Figures 2 and 3, we considered the errors in the model which can



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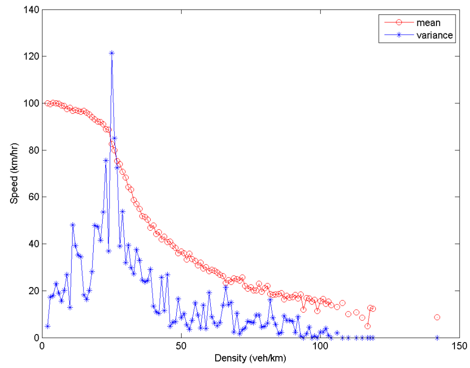

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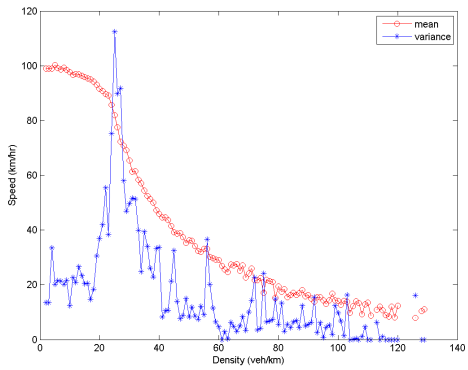

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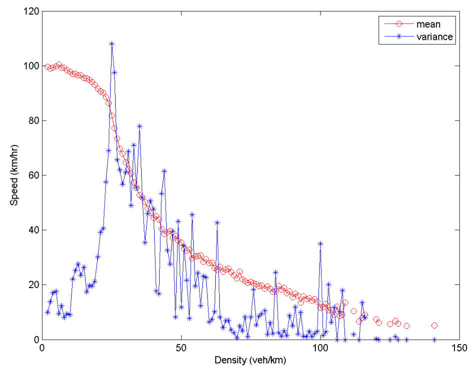

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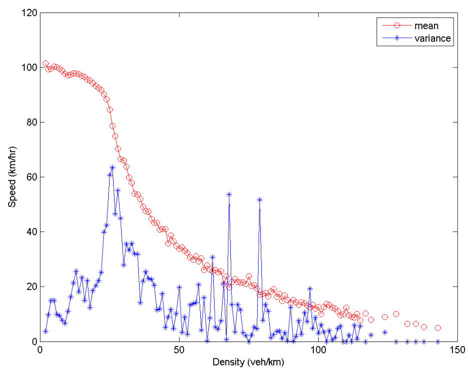

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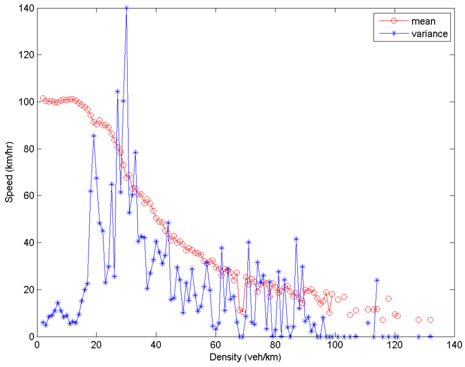
FIGURE 2 Weekday change of traffic speed variance from one-year observations at station 4000026 **with time aggregation level** 5 **minutes.**



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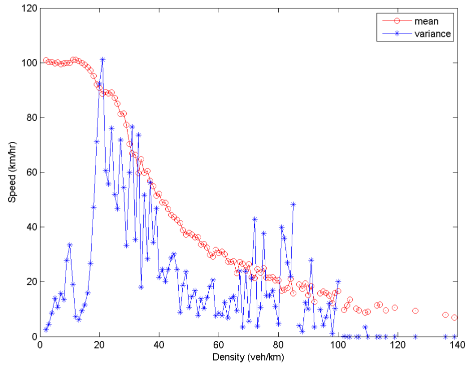

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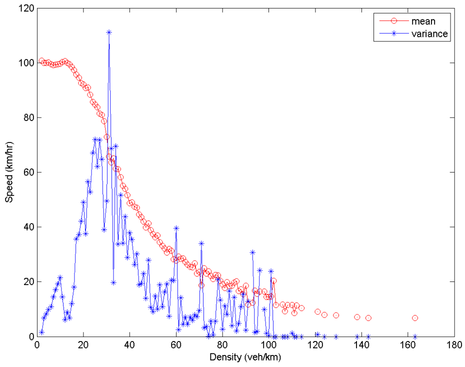

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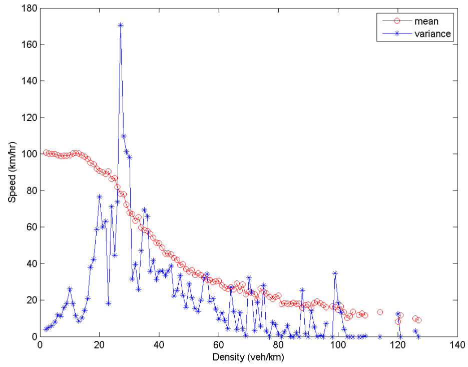

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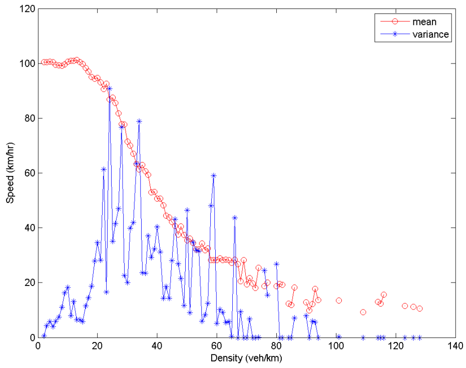

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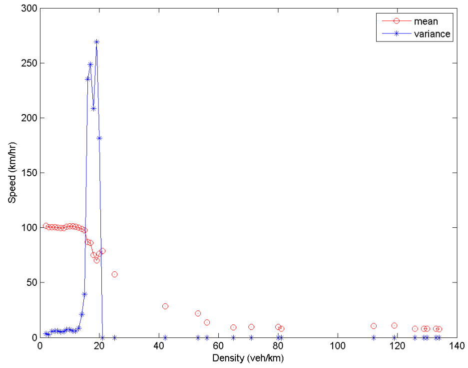
FIGURE 3 Weekday change of traffic speed variance from one-year observations at station 4001118 with time aggregation level 5 minutes



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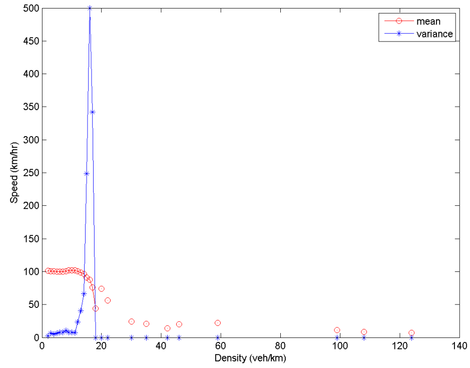

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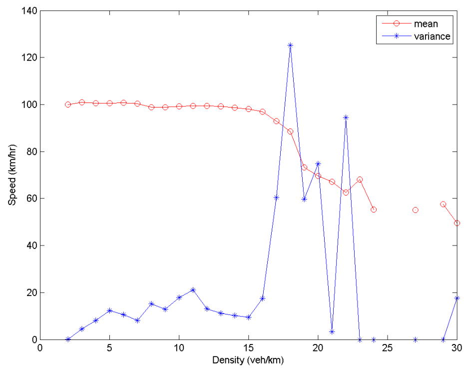

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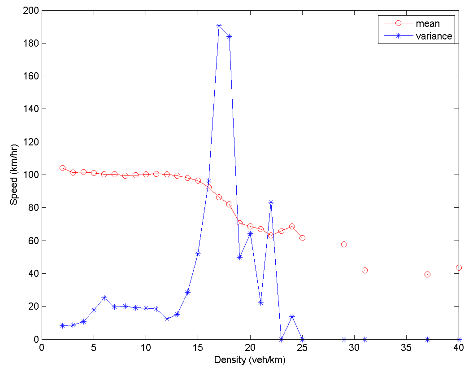

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FIGURE 4 Weekend change of traffic speed variance from one-year observations with time aggregation level 5 minutes

be described by

$$\varepsilon = V(k) - v(k, \theta) \tag{1}$$

in which $V(k)$ is the empirically observed traffic speeds, $v(k, \theta)$ is the speeds given by a speed-density model. For each value of density k, the model can be rewritten as

$$V_{il}(k) = v(k_i, \theta) + \varepsilon_{il} \tag{2}$$

with l varying from 1 to n_i (n_i is the number of speed observations over a long time period under density k_i) and i from 1 to j (k_j is jam density). For each value of i, the empirical variance of speed can be calculated by

$$s_i^2 = \frac{1}{n_i} \sum_{l=1}^{n_i} (V_{il} - V_{i*})^2 \tag{3}$$

with $V_{i*} = \frac{1}{n_i} \sum_{l=1}^{n_i} V_{ij}$. From the empirical traffic speed-density data as shown in Figures 2 and 3, the variance of traffic speeds is apparently heterogeneous, so we let $Var(\varepsilon_{il}) = \sigma_i^2$. The model is given by the following

$$V_{il}(k) = v(k_i, \theta) + \sigma_i^2 \tag{4}$$

in which $E(\varepsilon_{il}) = 0$, where $l = 1, \ldots, n_i$; $i = 1, \ldots, j$; and total number of observations equal $n = \sum_{i=1}^{j} n_i$. The choice of a specific speed-density model $v(k_i, \theta)$ is provided in Section 2.3. ε_{il} are independent Gaussian random variables. ε is, by construction, a random error equal to the discrepancy between empirical traffic speed observation $V(k)$ and traffic speed estimated from a speed-density model $v(k, \theta)$. θ is a vector of p parameters $\theta_1, \theta_2, \ldots, \theta_p$.

As aforementioned, let σ_i^2 be the traffic speed variance of ε_{il}. The values of σ_i^2, or their variances as functions of traffic density k_i, are unknown and has to be approximated. Generally, there are two cases in the difference of traffic speed variances: (1) the variance $\sigma_i - \sigma_{i+1}$ is small which means the traffic speed variance is relatively stable as traffic density varies. (2) the variance $\sigma_i - \sigma_{i+1}$ is large which indicates a large variation of traffic speeds at varying traffic densities. If the traffic speed variance $\sigma_i - \sigma_{i+1}$ is small, we feel confident to approximate the variances with homogeneity by assuming $Var(\varepsilon_{ij}) = \sigma^2$. In the case of a large traffic speed variation $\sigma_i - \sigma_{i+1}$, the physical interpretation is that various driver groups (novice/skilled, male/female, timid/aggressive) behave differently under changing traffic conditions. The empirical evidences as shown in Figures 2 and 3 is apparently against the assumption of homogeneous variances. In this case, the real heterogeneous variation of σ^2 is approximated by a variance function f such that $Var(\varepsilon_{ij}) = v(k_i, \sigma^2, \theta, \alpha)$. In this case, the variance function f is assumed to be dependent on the speed-density relationship represented by $v(k, \theta)$ in which speed is expressed as a function of traffic density plus a parameter set. For example, $v(k_i, \delta^2, \theta, \alpha) = \sigma_2 v(k_i, \theta)^\alpha$, where α is a set of parameters that have to be estimated or assumed to be known already. The estimation of the parameter set can be done through a least-square algorithm by minimizing the distances between the empirical traffic speeds and speeds estimated from a model. We usually can simplify the necessary assumptions by assuming that the vector θ_p varies in the interior of an interval. The function $v(k_i, \theta)$ is assumed to be twice continuously differentiable with respect to the parameters θ [1] [2].

2.2 Parametric modeling of the traffic speed variance

A variance function needs to be determined in order to estimate the heterogeneous traffic speed variance. To make the choice of variance function, some qualitative or quantitative indications of empirical traffic speed variance are needed [1]. Figure 6 plots the empirical mean of speed-density observations over one year and its corresponding speed variance. From Figure 6, we see that the variance of the empirical observations first grows as traffic density increase and then decreases with the maximum achieved at an intermediate density around the critical density $k_c(35 \rightarrow 45)$ (veh/km), which can be depicted by a parabola.

For an increasing variance, there are essentially two scenarios. The first scenario is that the variance varies as a power of the response

$$\sigma_i^2 = \delta^2\phi(k_i,\theta,\alpha) = \delta^2 v(k_i,\theta)^\alpha \tag{5}$$

The other one is that the variance varies as a linear function of the response

$$\sigma_i^2 = \delta^2\phi(k_i,\theta,\alpha) = \delta^2(1+\alpha v(k_i,\theta)) \tag{6}$$

Judging from the empirical observations of traffic speed variance in the previous section, it is found that these two variance functions are not appropriate to model the traffic speed variance with a parabola shape.

For a variance function varying like a parabola as shown in Figures 2 and 3, the most generalized function to model the structured speed variance is given by

$$\sigma_i^2 = \delta^2 + \delta^2\alpha_1(v_{max} + \alpha_2 - v(k_i,\theta))(v(k_i,\theta) - v_{min}) \tag{7}$$

in which v_{max} is the maximum value and v_{min} is the smallest value of $v(k_i,\theta)$. Let $v_{min} = 0$ and the maximum traffic speed v_{max} be the free-flow speed v_f, the variance function for the empirical traffic speed variance is given by

$$\sigma_i^2 = \delta^2(1.0 + \alpha v(k_i,\theta)(v_f - v(k_i,\theta))) \tag{8}$$

in which δ and α are parameters with physical meanings, $v(k,\theta)$ is open to all the existing single-regime speed-density models listed in [15] such as the Greenshilds model. A slight change to this model can be made by replacing the free flow speed term v_f with a highway design speed which is relatively higher v_f, here called v_d, will yield the following model

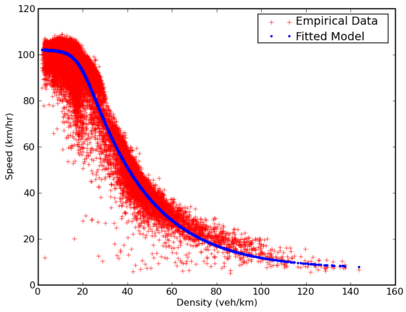
$$\sigma_i^2 = \delta^2(1.0 + \alpha v(k_i,\theta)(v_d - v(k_i,\theta))) \tag{9}$$

2.3 Choice of speed-density curves and corresponding variance functions

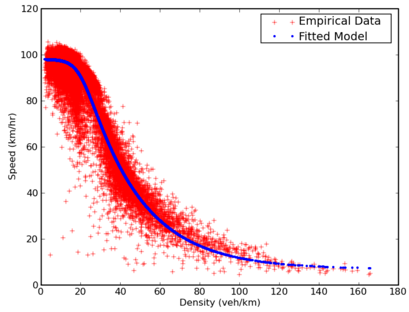
The proposed function to model the empirical traffic speed variances takes a functional form as can be seen from Equations (8) and (9) which in nature is a function of the speed-density model with two additional parameters δ^2 and α. Thus, the choice of a specific speed-density function apparently affects the performance of the variance function.

The empirical speed-density observations exhibit a reversed 'S' shape which makes logistic modeling a candidate. Therefore, a sigmoidal shape logistical speed-density function was proposed to model the speed-density curve [2]. The logistic speed-density model has a varying number of parameters from five to three which results in three distinct traffic speed variance functions with varying performances as can be seen from Figure 5. The most general five-parameter logistic speed-density model (5PLSDM) and the corresponding traffic speed variance function take a functional form of

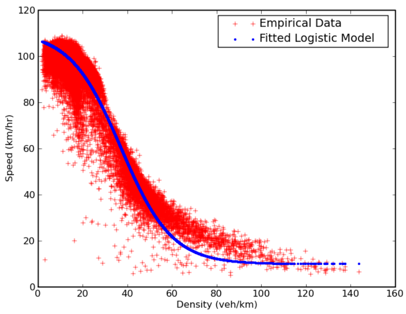
$$\begin{cases} v(k,\theta) = v_b + \frac{v_f - v_b}{(1+\exp(\frac{k-k_t}{\theta_1}))^{\theta_2}} \\ \sigma_i^2 = \delta^2(1.0 + \alpha v(k,\theta)(v_f - v(k,\theta))) \end{cases} \tag{10}$$



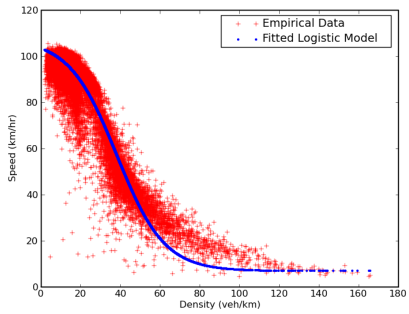
(a) 5PLSDM/4001118



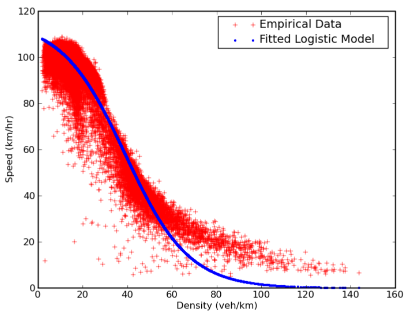
(b) 5PLSDM/4001119



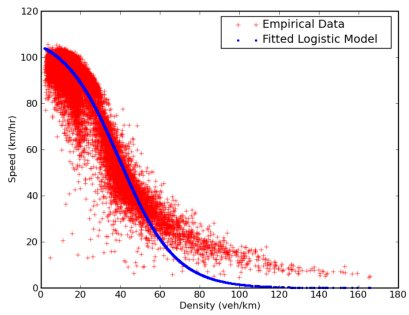
(c) 4PLSDM/4001118



(d) 4PLSDM/4001119



(e) 3PLSDM/4001118



(f) 3PLSDM/4001119

FIGURE 5 Performance of the logistic speed-density models fitted to the same set of empirical data from station 4001118 and 4001119

We found that the logistic speed-density function describes the empirical data [2]. This is the most general five-parameter logistic speed-density model (5PLSDM) in sigmoidal shape. In which, v_f and v_b are the upper and lower asymptotes respectively. Specific to our case, v_f represents free flow speed. v_b is the average travel speed under stop and go conditions. This parameter assumes that traffic has finite movements even in congested situations [16]. θ_1 is a scale parameter which describes how the curve is stretched out over the whole density range, and θ_2 is a parameter which controls the lopsidedness of the curve. The parameter k_t is the turning point at which the speed-density curve makes the transition from free-flow to congested flow.

A four-parameter logistic speed-density model (4PLSDM) is obtained by reducing the parameter θ_2 and its corresponding variance function is given by

$$\begin{cases} v(k,\theta) = v_b + \frac{v_f - v_b}{1+\exp(\frac{k-k_c}{\theta_1})} \\ \sigma_i^2 = \delta^2(1.0 + \alpha v(k,\theta)(v_f - v(k,\theta))) \end{cases} \tag{11}$$

The physical meaning of the other parameters remains unchanged. Different from the 5PLSDM, the 4PLSDM captures the critical traffic density k_c instead of k_t. The three-parameter logistic speed-density model (3PLSDM) can be obtained by removing the user-specified average travel speed at stop-and-go traffic conditions and the variance function based on this 3PL curve is given by

$$\begin{cases} v(k,\theta) = \frac{v_f}{1+\exp(\frac{k-k_c}{\theta_1})} \\ \sigma_i^2 = \delta^2(1.0 + \alpha v(k,\theta)(v_f - v(k,\theta))) \end{cases} \tag{12}$$

The generalized four-parameter logistic speed-density model (asymmetric sigmoidal function) results from the integration of the differential equation

$$\frac{dv}{dk} = cv[1 - (\frac{v}{v_f})^\gamma] \tag{13}$$

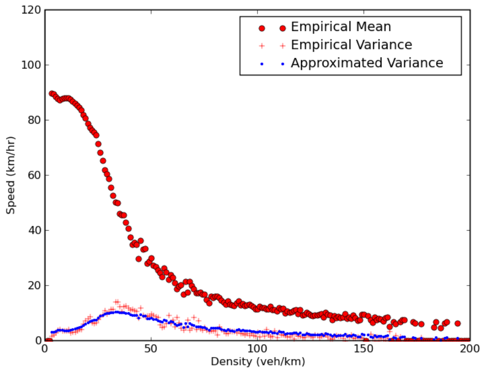
which mimics the form of the so-called Nelder's model [17]. The constant v_f is the free-flow speed, and the constant $\gamma > 0$ is the asymmetry coefficient, when $\gamma = 1$, the equation (13) turns into the well-known logistic model [18]. Similarly, the three-parameter logistic speed-density model can be obtained from the differential equation (13) by setting $\gamma = 1$

$$\frac{dv}{dk} = cv(1 - \frac{v}{v_f}) \tag{14}$$

Equation (14) has been found to be a more meaningful form of the logistic function in describing the growth pattern dynamics [19] such as plant and population growth. The performance of 5PL, 4PL, and 3PL is referred to Figure 5, interested readers can find more detailed information regarding the logistic modeling of speed-density relationship in [2]. In this paper, we only considers the performance of the variance function in which the five-parameter logistic speed-density model is applied. For the other two cases, they are just minor changes of model parameters.

3 Calibration of Model Parameters

As aforementioned, the modeling of traffic speed variance is dependent on the speed-density relationship. Figure 6 shows the performance of the proposed variance function with a five-parameter logistic speed-density model when compared to empirical data, instead of plotting the variance curve, this figure plots the standard deviation of traffic speed and the same from the variance function. It is found that the proposed variance function tracks empirical variance faithfully. Table 1



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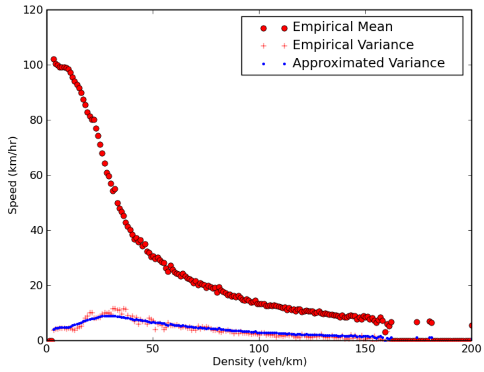

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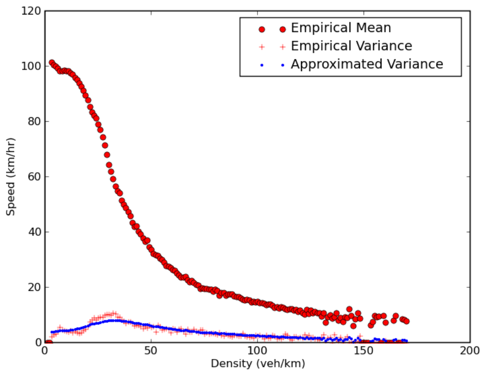
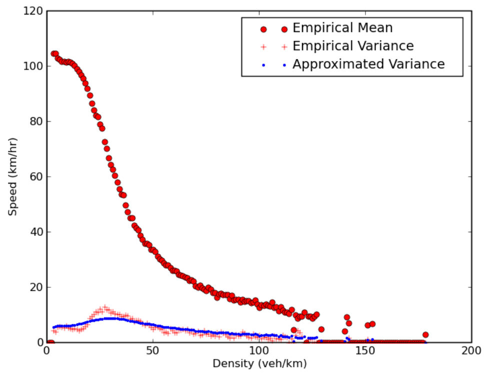


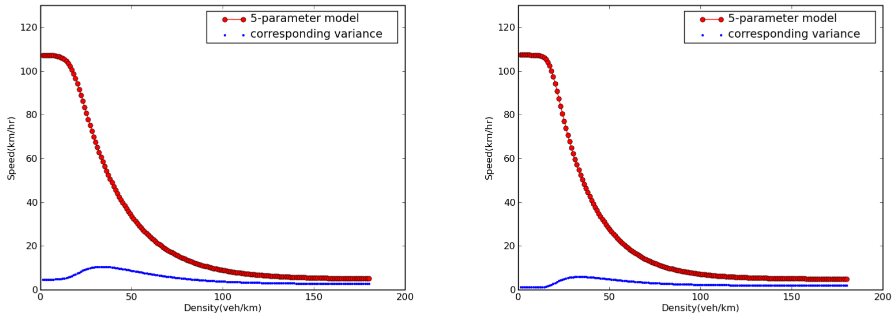
(c) 4001138



(d) 4001139

FIGURE 6 Approximated standard deviation (std) plotted against empirical std

listed the optimized model parameters for the proposed variance function. The model parameters in the speed-density model are obtained through an iterative least-squares procedure while the two additional parameters in the variance function are obtained by a maximum likelihood estimation method since we assume the error term ε_i is a Gaussian variable. The quasi-likelihood estimation is provided for those who have particular reason to question the Gaussian assumption. For more details about the statistical estimation techniques, interested readers are referred to [1]. From the magnitude of the estimated parameters particularly δ^2 and α, we observe that δ^2 is relatively stable while α suffers a large variation. The existence of a constant term in the variance model, in this case δ^2, can be explained in both practical and theoretical ways. The physical meaning of δ^2 is the maximum possible variance when traffic density is nearly 0 (corresponds to free flow condition). This implies the fact that drivers from different driver groups (aggressive or cautious, old or young) have their own preferred free flow speed. And this location-specific parameter is dependent on empirical data. To frame it in a more theoretical sense, the existence of this parameter can be



(a) (b)

FIGURE 7 Performance of the variance function with a five-parameter logistic speed-density model different free flow speed: (a) $\delta^2 = 1.3$**,** $\alpha = 0.002$ **,** $v_d = 120(km/hr)$ **and (b)** $\delta^2 = 1.2033$**,** $\alpha = 0.0014$**,** $v_f = 107.44(km/hr)$

verified by a likelihood ratio test by expressing the hypothesis as $\{\delta^2 = 0\}$, the results signify a better fit when $\{\delta^2 \neq 0\}$.

4 Applications of Speed Variance Models

Most of the identified applications of a traffic speed variance model points to a safety side such as how fatality rate and crash rate is a function of speed variance and other factors. For example, Lave [5] found that speed variance has more effect on fatalities than traffic speed does in which fatality rate is a function of traffic speed, speed variance and other factors. For more information regarding such a relationship, interested readers are directed to [5] for details. For a relationship between freeway speed variance with lane changing behavior, or with vehicle heterogeneity, interested readers are referred to [4]. Different from the applications aforementioned, this paper tries to explore how a speed variance model can help estimate travel time and its variability.

4.1 Simulation Test Bed for Validation of the Speed Variance Model

The travel time variance model, which is developed as an important application of the proposed speed variance model, needs to be validated before it can be applied in practice to predict the range of travel time for freeway traffic information management. Due to the lack of field-collected travel time data of individual vehicles, microscopic simulation tool VISSIM is used in this research to generate both detector data and vehicle's travel time data. Considering that the travel time variance model is a general model which is expected to be applied to any types of freeways, an existing urban freeway simulation model provided along with the installation of VISSIM is directly used as the simulation test bed for this research.

As illustrated by Figure 8, the simulation model represents a 3.1-mile segment of I-405 in Redmond, Washington. The detectors are placed near the entrance of the northbound approach, which is noted by the green dot in Figure 8. The travel time section begins at the location of the

TABLE 1 Optimized parameters for the traffic speed variance function and the five-parameter logistic speed-density model (speed v_f in km/hr, density k_t in veh/km), S represents Stations, 01-39 is abbreviated from 4001101 to 4001139, 25-63 is abbreviated from 4000025 to 4000063

S	δ^2	α	v_f	k_t	θ_1	θ_2	S	δ^2	α	v_f	k_t	θ_1	θ_2
01	1.2	0.0014	107.44	17.53	1.8768	0.0871	25	1.2	0.005	96.09	20.04	3.6202	0.1323
02	1.2	0.004	99.92	16.12	2.1098	0.0947	26	1.3	0.003	99.93	20.34	3.0470	0.1269
03	1.4	0.010	106.89	14.40	1.7388	0.0714	27	1.3	0.04	96.14	22.89	4.5292	0.1941
04	2.4	0.008	47.52	32.61	0.1188	0.1835	28	1.3	0.006	99.88	14.30	0.2418	0.0106
05	1.3	0.004	86.57	24.39	1.0094	0.0401	29	1.1	0.003	106.80	16.95	2.4325	0.1059
06	1.6	0.003	92.71	21.72	3.9212	0.1835	30	1.6	0.005	91.04	22.21	3.2091	0.1138
07	1.4	0.002	99.39	21.26	3.8762	0.1928	31	1.8	0.002	88.99	28.77	5.1484	0.1499
08	1.5	0.013	95.06	20.33	3.15	0.1628	32	1.3	0.003	97.05	19.61	2.2104	0.0746
09	1.1	0.002	111.06	17.01	2.5501	0.1074	33	1.4	0.002	95.69	22.43	3.0685	0.1249
10	1.3	0.012	96.16	19.03	2.0220	0.0938	34	1.3	0.007	98.05	22.24	4.4141	0.1688
11	1.3	0.009	97.64	17.52	2.2787	0.0899	35	1.1	0.005	107.96	21.24	4.1009	0.1736
12	1.2	0.008	100.67	12.63	2.0386	0.0899	36	1.4	0.009	101.92	21.67	3.7766	0.1478
13	1.3	0.006	103.02	15.52	2.0674	0.0857	37	1.3	0.008	98.47	21.07	3.7207	0.1326
14	1.3	0.010	98.97	20.20	3.1219	0.1179	38	1.2	0.005	106.31	19.42	4.6129	0.1802
15	1.3	0.009	98.60	17.69	3.0240	0.1202	39	1.1	0.006	110.36	17.44	3.8358	0.2096
16	1.2	0.004	105.51	16.48	3.3903	0.1404	40	1.2	0.008	108.57	16.86	2.6591	0.1181
17	1.3	0.006	99.35	13.25	1.8755	0.0926	41	1.3	0.004	105.64	19.89	3.3156	0.1181
18	1.3	0.004	102.12	18.99	3.34	0.1231	42	1.3	0.008	105.40	18.67	3.8394	0.1387
19	1.4	0.007	98.08	19.97	3.53	0.1300	43	1.0	0.009	109.58	18.19	2.7535	0.1140
20	1.2	0.005	104.22	18.06	3.3054	0.1110	44	1.4	0.015	99.68	19.64	2.7885	0.0995
21	1.4	0.008	100.06	19.22	3.3051	0.1189	45	1.3	0.009	101.67	18.83	2.7745	0.0985
22	1.3	0.007	100.06	19.22	3.3051	0.1189	46	1.1	0.005	108.45	17.83	2.6356	0.1199
23	1.3	0.010	97.45	20.98	4.9820	0.1901	47	1.2	0.008	108.24	15.25	2.4505	0.1259
24	1.1	0.003	114.36	17.55	4.7015	0.1901	48	1.3	0.006	101.68	21.37	4.5757	0.1254
25	1.5	0.013	89.34	17.42	5.3515	0.2271	49	1.4	0.014	96.72	17.79	5.2587	0.1696
26	1.2	0.010	110.55	12.29	2.0450	0.0714	50	1.7	0.016	87.05	21.47	3.5007	0.1060
27	1.4	0.012	99.11	22.67	5.3573	0.1994	51	1.5	0.008	95.37	16.70	2.4823	0.1341
28	1.4	0.009	98.08	28.67	6.61	0.4005	52	1.3	0.012	102.50	15.05	1.7244	0.0810
29	1.3	0.008	104.20	22.27	4.82	0.1787	53	1.3	0.008	99.22	17.10	1.7835	0.0716
30	1.3	0.008	105.09	24.04	5.5045	0.2693	54	1.5	0.012	89.83	17.75	1.8373	0.0623
31	1.5	0.007	97.72	23.90	5.1731	0.2009	55	0.9	0.005	137.75	15.72	1.2974	0.0512
32	1.6	0.012	95.57	22.53	4.4708	0.1535	56	1.3	0.006	97.12	15.72	1.2974	0.0512
33	1.9	0.015	72.32	21.28	1.2734	0.0341	57	1.6	0.008	87.67	16.91	1.2206	0.0512
34	1.5	0.018	92.66	21.17	5.0568	0.1681	58	1.4	0.009	94.76	16.73	1.8321	0.0618
35	1.2	0.016	103.94	11.24	2.7827	0.0653	59	1.2	0.014	102.27	13.37	1.5524	0.0650
36	1.8	0.008	88.81	19.97	3.8557	0.1687	60	1.4	0.010	91.08	19.45	2.0860	0.0592
37	1.4	0.013	101.40	14.28	3.7376	0.1380	61	1.3	0.012	91.40	20.18	2.6633	0.0911
38	1.3	0.009	99.64	18.22	3.67	0.1413	62	1.3	0.008	94.35	15.14	1.8434	0.0604
39	1.4	0.011	102.99	17.87	3.45	0.1410	63	1.2	0.006	108.43	13.53	1.1753	0.0450

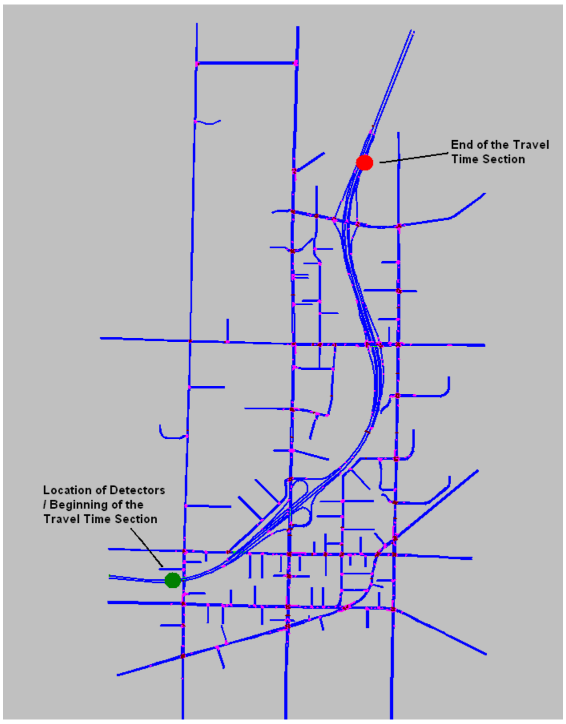


FIGURE 8 The Vissim simulation test bed

detectors and ends at the end of the northbound approach as noted by the red dot in Figure 8. There are three lanes at the location of detectors and four lanes at the end of the travel time section. Before running the simulation, most parameters of the simulation remain unchanged, except that the free-flow 85^{th} percentile speed is set to 68 mph rather than the default value of 55 mph. Meanwhile, traffic volumes are dynamically assigned at different time periods throughout the simulation. Totally, 10 simulation runs are conducted using different random seeds while each simulation lasts 4500 seconds.

VISSIM outputs the aggregated detector data including the average speed and the average count of vehicles for every 60 seconds. It also outputs each individual vehicle's travel time when the vehicle passes the end point of the travel time section. However, VISSIM does not have the function to associate individual vehicle's travel time with the detector data of the time period when the vehicle passes the detectors. In order to realize this, a data preprocessing has been conducted with the help of SQL and ACCESS database. As a result, the average volume and speed of the time period when the individual vehicle passes the detector is finally linked with the vehicle's travel time, which provides the ground truth data support for validating the travel time variance model. To better verify the validity of the VISSIM simulation model, a fundamental flow-density and speed-density relationship is plotted from the VISSIM output data as can be seen in Figure 9.

4.2 Results Analysis

To facilitate the validation of the proposed traffic speed variance model, the authors tried two approaches to generate the travel time and travel time variation: (1) The first one is through a

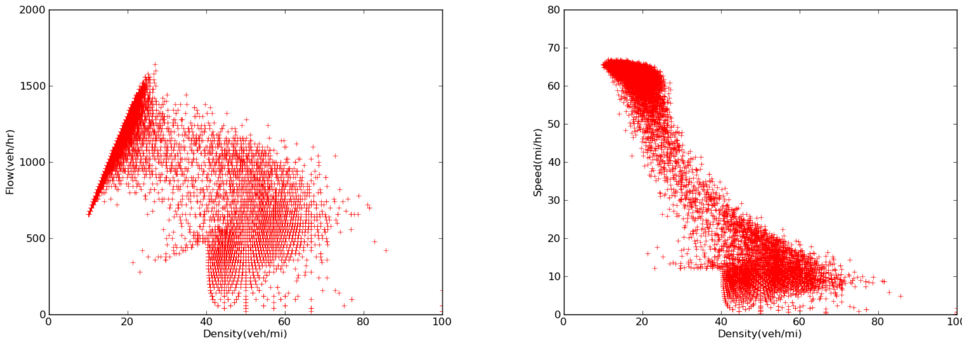
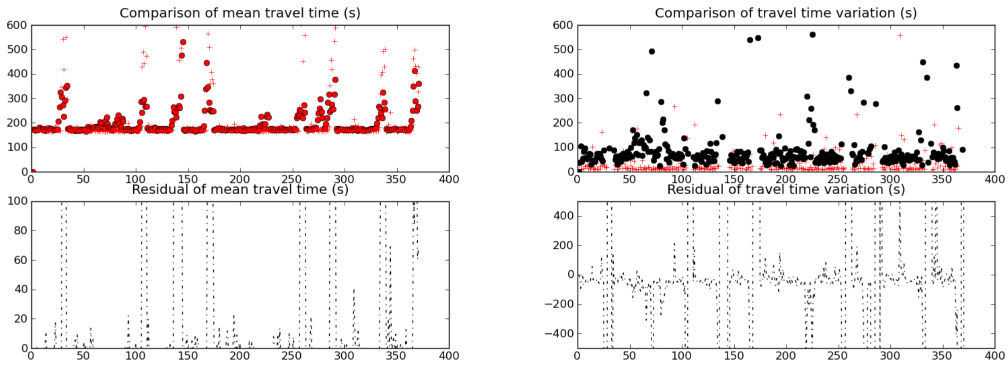


FIGURE 9 The fundamental flow-density and speed-density relationship generated from VISSIM simulation data

VISSIM simulation model (or simulation) in which the highway and vehicle characteristics are specified. For a given length of highway segment, each individual vehicle's travel time is recorded so that the mean travel time and travel time variability can be calculated. (2) The alternative way is through the traffic speed variance model (or model), as aforementioned, the proposed traffic speed variance model is a function of the speed-density response curve. Particular to this application, we used the five-parameter logistic speed-density model and its corresponding traffic speed variance curve. To be specific, the mean travel time estimated from the model is obtained by dividing the length of highway segment over the mean travel speed given by the five-parameter logistic speed-density model. The travel time variation are captured through the proposed traffic speed variance model; for any given traffic density, the traffic speed variance at each density can be obtained from the proposed variance function thus the travel time variation can be estimated. In nature, no matter the traffic speed or the speed variance is a function of traffic density associated with additional parameters such as the free-flow speed, the stop-and-go speed at congested conditions, the scale and shape parameters in both the speed and the speed variance function. The traffic density value from the simulation VISSIM simulation model is recorded for use in the traffic speed variance model to estimate travel time variability.

A quick comparison of the mean travel time from the VISSIM simulation model and from the traffic speed variance model is demonstrated in Figure 10, the residual of the two mean travel time is plotted underneath as well. Figure 10(a) shows that the mean travel time from both the VISSIM simulation model and the traffic speed variance model matches with each other for most of the samples, but indeed there are some travel times showing a larger gap which can be statistically treated as outliers. In addition, the comparison result is also dependent on the choice of the model parameters such as $v_f, v_b, k_t, \theta_1, \theta_2, \alpha, \delta$. Most of the model parameters are physically meaningful and can be calibrated from empirical observations. Table 1 shows the calibrated parameters from the empirical observations on George state route 400 in the year of 2003. The calibration and estimation of model parameters is not the focus of this paper, interested readers are directed to [1] [20] for details regarding *Levenberg-Marquardt algorithm* and *Maximam Likelihood Estimation* method. The comparison of the travel time variance from the VISSIM simulation model and the traffic speed variance model is shown in Figure 10(b). From the comparison, it is worth mentioning that the travel time variance estimated from the traffic speed variance model is some-



(a) Mean Travel Time Comparison (b) Travel Time Variation

FIGURE 10 Comparison of the mean travel time ("+" representing the proposed model result and the dot is simulation result, the same for variances) and travel time variation from simulation and the speed variance model

how affected by the choice of model parameters, but this parameter-dependency does not degrade the potential benefits of the proposed traffic speed variance model. On the contrary, the choice of suitable model parameters gave the users flexibility in the application domain. In general, this simple illustrative example indicates that the traffic speed variance model is functional in terms of estimating mean travel time and its variability.

5 Conclusion and future remarks

This paper proposed a generalized variance function to model empirical traffic speed variance. The variance function captures the nonlinear and heterogeneous nature of a parabolic shaped variance. The variance function has two features: it is dependent on the speed-density curve and it contains two additional parameters which have to be set either as constants for simplification or to be estimated from empirical data.

The major findings of this research are:

1. The structured traffic speed variance is the results of naturally occurring macroscopic traffic conditions. The empirical variance takes a parabolic shape which first increases to a local maximum and then decreases as traffic density increases.

2. The pattern of structured traffic speed variance is different between weekdays and weekends. It is found that this pattern is consistent on either weekdays (from Monday to Friday) or weekends (Saturday and Sunday), but this proposed speed variance model works better for weekdays than weekends data.

3. A parametric traffic speed variance function is used to model traffic speed variance in terms of its nonlinearity and heterogeneity. The model parameters are calibrated through empirical data.

4. The proposed variance function match the empirically observed traffic speed variances. In particular, the five-parameter logistic speed-density model and its corresponding speed variance function describes the empirical variance accurately.

5. The application example shows that the proposed traffic speed variance model is capable of estimating the travel time and its variability which has been verified by the designed VISSIM simulation example.

References

[1] M.-A. Poursat E. Jolivet. S. Huet, A. Bouvier. *Statistical Tools for Nonlinear Regression A Pracital Guide With S-PLUS and R Examples, Second Edition*. Springer, 2004.

[2] Qian-Yong Chen Haizhong Wang, Jia Li and Daiheng Ni. Logistic modeling of equilibrium speed-density relationship. *Transportation Research Part A: Policy and Practice*, 45:554–566, July 2011.

[3] Qian-Yong Chen Haizhong Wang, Daiheng Ni and Jia Li. Stochastic modeling of equilibrium speed-density relationship. *Journal of Advanced Transportation*, 2011.

[4] S. Park and S. G. Ritchie. Exploring the relationship between freeway speed variance, lane changing, and vehicle heterogeneity. In *The* 83^{rd} *Annual Meeting of Transportation Research Board*, 2004.

[5] Charles A. Lave. Speeding, coordination, and the 55 mph limit. *American Economic Review*, 75(5):1159–1164, December 1985.

[6] Gadiraju R. Garber, N. J. Factors affecting speed variance and its influence on accidents. *Journal of Transportation Research Record*, 1213:64–71, 1989.

[7] Gadiraju R. Garber, N. J. Impact of differential speed limits on the speed of traffic and the rate of accidents. *Journal of Transportation Research Record*, 1375:44–52, 1992.

[8] Fitzpatrick K. Bauer-K. M. Collins, J. and D. W. Harwood. Speed variability on rural two-lane highways. *Journal of Transportation Research Record*, 1658:60–69, 1999.

[9] Federal Highway Administration. *Speed Management Work Plan*. FHWA-National Highway Traffic Safety Administration, Washington DC, April 1997.

[10] Kyeong-Pyo Kang and Gang-Len Chang. A robust model for optimal time-of-day speed control at highway work zones. *IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS*, VOL. 7(1):115–123, MARCH 2006.

[11] Philip E. Graves, Dwight R. Lee, and Robert L. Sexton. Speed variance, enforcement, and the optimal speed limit. *Economics Letters*, 42(2-3):237–243, 1993.

[12] Saifallah Benjaafar, Kevin Dooley, and Wibowo Setyawan. Cellular automata for traffic flow modeling, 1997.

[13] Hesham Rakha and Wang Zhang. Estimating traffic stream space-mean speed and reliability from dual and single loop detectors. *Transportation Research Record*, 1925:38–47, 2005.

[14] Jian John Lu. Guideline development for minimizing operating speed variance of multilane highways by controlling access design. University of Tennessee, Knoxville.

[15] Haizhong Wang; Jia Li; Qian-Yong Chen; Daiheng Ni. Representing the fundamental diagram: The pursuit of mathematical elegance and empirical accuracy. In *TRB 89th Annual Meeting Compendium of Papers DVD*. Transportation Research Board, January 2010.

[16] J. L. Schofer J. S. Drake and A. D. May. A statistical analysis of speed-density hypotheses. *Highway Research Record*, 156:53–87, 1967.

[17] J. A. Nelder. The fitting of a generalization of the logistic curve. *Biometrics*, 17:89–100, 1961.

[18] Rudolf Scitovski Dragan Jukic. The existence of optimal parameters of the generalized logistic function. *Applied Mathematics and Computation*, 77:281–294, 1996.

[19] G. Hardin. A more meaningful form of the textquotedblleft logistic" equation. *The American Naturalist*, 79:279–281, 1945.

[20] Marko Ledvij. Curve fitting made easy. *The Industrial Physicist*, 2003.