ESTIMATING CRITICAL GAP - A COMPARISON OF METHODOLOGIES USING A ROBUST, REAL-WORLD DATA SET

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ABSTRACT

Understanding the gap acceptance behavior of drivers is critical to transportation professionals dealing with roadway design and safety. Inaccurate information on how drivers utilize gaps in traffic can lead to inappropriate and potentially dangerous design decisions. Arguably, the best mechanism for understanding this behavior is through field investigations; however, little uniformity exists regarding best practices for analyzing gap acceptance field data.

A number of different methodologies have been proposed and are currently in use for analyzing gap acceptance data. Subsequently, the question that arises is whether the analysis method chosen affects the results of the analysis. Previous works have made comparisons of different analysis methods, but with the intent of demonstrating the superiority of the author's new analysis method. The research conducted herein is focused on a direct, and objective comparison of existing methodologies.

More specifically, this paper uses observations from a large-scale-field study of 2,700 drivers, and presents a comparison of the five most commonly employed methods with two variations of each for a total of ten unique gap analysis methods. The lone criteria for each analysis method considered was that it have a firm fundamental base and be computationally simple enough for everyday application. The ease of implementation, sample size requirements, and results of each method are discussed.

Methods used for analysis resulted in significantly different results. This raises concerns when comparing studies using different analysis methodologies. In addition, critical gap estimates from the evaluated methodologies were compared with the widely accepted values of the Highway Capacity Manual.

Keywords: Gap acceptance, critical gap, gap analysis

INTRODUCTION

In the field of transportation safety it is well understood that crashes can be attributed to failures of the road, the vehicle, the user, or some combination thereof. One common driving task that encompasses each of these elements occurs when drivers are tasked with making a gap acceptance decision either merging into or crossing a lane of traffic.

Given that "driver error" is cited as a contributing factor in 93 percent of all crashes, understanding driver behavior is an essential element in mitigating the crash problem (1). Among the more dangerous roadway elements are unsignalized intersections where driver behavior is directly related to operational and safety performance (1). More specifically, drivers' gap acceptance decisions have serious consequences, and in many situations, the result of a poor gap acceptance decision is a crash.

The process of a driver's gap acceptance decision is influenced by an individual's goals and attitudes and is affected by stimuli from their surroundings. It is widely accepted that the best method of observing naturalistic driver behavior is through field investigation (2). However, there is little uniformity in how gap acceptance data collected through field investigation should be analyzed.

Problem Statement

A need exists to foster a greater understanding of drivers' gap acceptance behavior based upon real-world empirical data. Understanding this aspect of driver behavior is critical to transportation professionals dealing with roadway design and safety.

Inaccurate or incorrectly used information regarding how drivers utilize gaps in traffic can lead to inappropriate design decisions. If overly conservative gap acceptance behavior is assumed (large critical gap), roadway elements will be overdesigned thus wasting money, compromising efficiency, and possibly having deleterious effects on other elements of the roadway system. If overly aggressive gap acceptance behavior is assumed (small critical gap), the results will be a design that has insufficient capacity for turning movements and can even force drivers to make risky gap acceptance decisions. Having access to a more accurate estimate of critical gap that correctly reflects the conditions under which it is be applied would lead to safer and more efficient roadway design.

One of the greatest challenges of developing an accurate estimate of critical gap is the lack of uniformity in how gap acceptance data is analyzed and how the critical gap is estimated.

Research Objective

Based upon the existing research needs and the potential for utilizing data collected using a newly developed data collection tool, an overarching goal of this research effort was to identify appropriate methods for estimating critical gaps across a series of variables. A specific objective of this research paper was to:

> *Compare different methods of analyzing gap acceptance data to understand the impact of methodological selection on measures such as critical gap.*

STUDY DESIGN AND METHODOLOGY

In an effort to achieve the established research objective a large-scale field study was completed by over a dozen team members in Massachusetts and Oregon. In total 60 sites, 2,767 drivers, 10,419 driver decisions, and 22,639 gaps in traffic were observed. The observations were focused on left and right turning maneuvers at unsignalized T-intersections. The data was collected over the course of a year varying both day of week (weekdays only for this phase of analysis) and time of day (daylight only due to visibility requirements). These observations represent a wide array of site conditions, under various traffic conditions, by many different drivers.

The field study utilized a newly developed program that can be operated by one person on a laptop computer in the field. A second observer is required if detailed vehicle and driver characteristics are to be simultaneously collected, which was done during the field study relating to this research initiative. To ensure that the results of the field study were accurate, a prior video validation was performed (3).

Once gap acceptance data has been compiled there remains a myriad of methods by which overall analyses of gap acceptance, and critical gap analyses in particular, can be completed. As part of this research initiative a number of different methods were used to determine the critical gap. The resulting critical gaps derived from each method were then compared. When determining the overall utility of each method, characteristics such as ease of use, required sample size, and required site conditions were taken into consideration.

As part of this objective, the results of the different analysis methods were compared to the standard values reported in the *Highway Capacity Manual 2010*. These values were adjusted, per adjustment factors detailed in the *Highway Capacity Manual 2010*, to reflect the conditions under which the data was collected.(4) Conclusions were drawn on how closely the numbers compare, and whether or not it would be advisable for the next version of the Highway Capacity Manual to consider more adjustment factors when determining critical gap.

RESULTS AND ANALYSIS

There are a number of different methods that have been proposed to analyze gap acceptance data. Some of these methods were eliminated from consideration in this research initiative because they were only applicable under certain traffic conditions. For example, the Siegloch (1973) method is only applicable under saturated conditions.(5) For most situations in the field, and all of those studied in this research initiative, these methods are not appropriate.

Other methods were eliminated because they were two too computationally demanding to be implemented for most reasonable studies. These methods involved iteratively solving multiple equations and do not provide closed solution sets. One such method, proposed by Troutbeck (1992), involves the principle of maximum likelihood analysis. This method has been approximated by less computationally complex mathematical models that were incorporated in some of the methods utilized.(6)

After eliminating methods that were inappropriate or impractical, five methods, each with two variations remained. The methods that were analyzed using the large data set collected in this research initiative are presented in **Table 1**.

TABLE 1 Comparison of Gap Acceptance Analysis Methods

Methods	Variation
Average Accepted Gap	All accepted gaps Accepted gaps < 12 seconds
Raff Method	All gaps All accepted gaps and maximum rejected gaps
Cumulative Acceptance	All accepted gaps Accepted gaps < 12 seconds
Equilibrium of Probabilities	All gaps All accepted gaps and maximum rejected gaps
Fit Maximization	All gaps All accepted gaps and maximum rejected gaps

Details on each of the methods used are discussed in following sections and the results are then compared between the methods.

Average Accepted Gap Method

This method is the most computationally simple of all the methods, however it is the only method that does not provide an estimate of critical gap. The average accepted gap is often used as a proxy for critical gap to allow for comparison of different data sets or the effects of different characteristics.

Implementation

To employ this method the accepted gaps are tabulated and then averaged. With the second variation, accepted gaps over 12 second are excluded from analysis. The rationale behind this variation is that gaps in traffic over 12 seconds are universally accepted by drivers and therefore do not represent true gap acceptance decisions.

Sample Size Requirements

Since this method only uses accepted gaps and not rejected gaps, a much large data set is required for reasonable conclusions to be drawn. The usable data from a sample is further reduced when gaps over 12 seconds are excluded, necessitating an even larger sample size for meaningful results.

Results

The Average Accepted Gap Method was employed to analyze the data from the field study. **Figure 1** presents the results for left and right turning maneuvers.

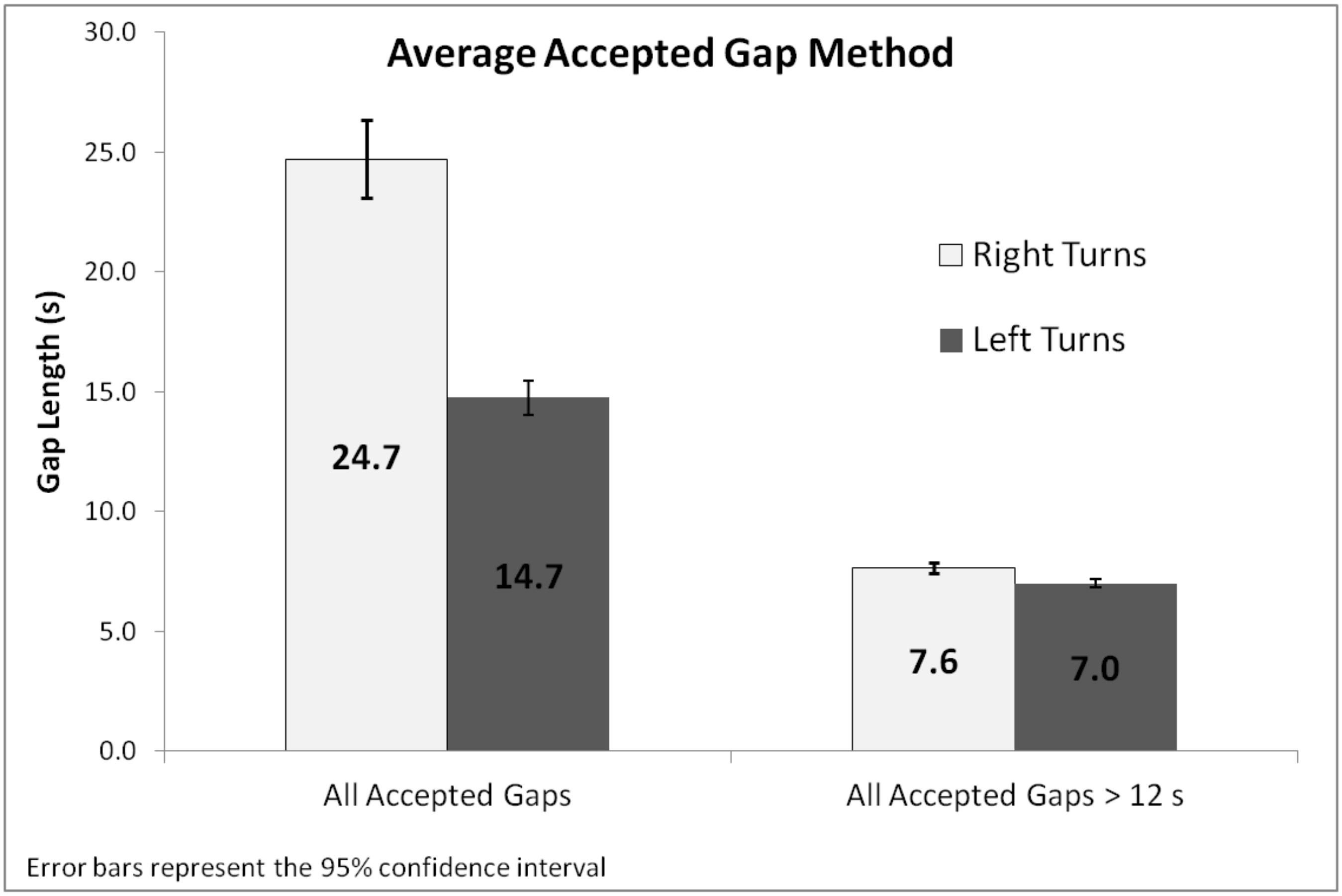

FIGURE 1 Results of Average Accepted Gap Method Analysis

As would be expected, excluding the gaps over 12 seconds significantly reduces the average accepted gap. With the gaps over 12 seconds excluded, the average accepted gap is relatively close to the critical gap estimated by the other methods utilized.

Overall, this method was useful in quickly presenting results that could be used to compare different data sets. However, since rejected gaps are not utilized in the analysis a considerable amount of available information on driver decision making is wasted by using this method. The biggest drawback of this method is that critical gap is not estimated. As this is an important metric in many applications, this is a significant shortcoming of this method.

Raff Method

One of the most commonly used analysis methods is the Raff Method. Proposed by Morton S. Raff in the late 1940's , this method is both conceptually logical and computationally simple.(7)

Implementation

To employ this method the accepted gaps and rejected gaps must be binned into set time intervals, such as 2 second intervals. For each interval the number of gaps accepted, number of gaps rejected, percent of gaps accepted, and percent of gaps rejected must be tabulated. So for any gap length bin, the reduced data will show the percent of gaps accepted and percent of gaps rejected. Such a table of reduced data is presented in **Figure 2**.

By graphing the resulting percent accepted and percent rejected the critical gap can be determined. By the Raff definition, the gap length where the percent of gaps rejected equals the percent of gaps accepted is the critical gap. This corresponds to the point where 50 percent of gaps are rejected and 50 percent of gaps are rejected. Assuming the sample is representative of the driving population this would also be the gap length where a driver has a 50 percent probability of accepting the gap.

The variation on this method is to consider just the maximum gap rejected by each driver, not all gaps rejected by each driver. This variation removes the potential bias towards passive drivers who reject many gaps before accepting one.

Gap Size	Total Rejected	Total Accepted	Count Rejected	Count Accepted	Percent Rejected	Percent Accepted
≤ 2	1015	15	1015	15	98.5	1.5
3	1220	45	205	30	87.2	12.8
4	1336	85	116	40	74.4	25.6
5	1408	137	72	52	58.1	41.9
6	1464	193	56	56	50.0	50.0
7	1487	259	23	66	25.8	74.2
8	1508	318	21	59	26.3	73.8
9	1515	374	7	56	11.1	88.9
≥10	31	1101	31	1101	2.7	97.3

FIGURE 2 Example of Raff Method Reduced Data

Sample Size Requirements

Since this method utilizes both accepted gap and rejected gap data, a smaller sample size will give more meaningful results. All driver choices are reflected in this method of analysis.

With the maximum rejected gap variation some of the collected data is not used, thereby necessitating a larger sample size for meaningful results.

Results

The Raff Method was employed to analyze the data from the field study, the results are shown in **Figure 3** along with the results for the maximum gap accepted variation. The bars represent the percentage values as tabulated and the lines are used to interpolate between values. The critical gap value was estimated to the nearest 0.5 second interval from the graph.

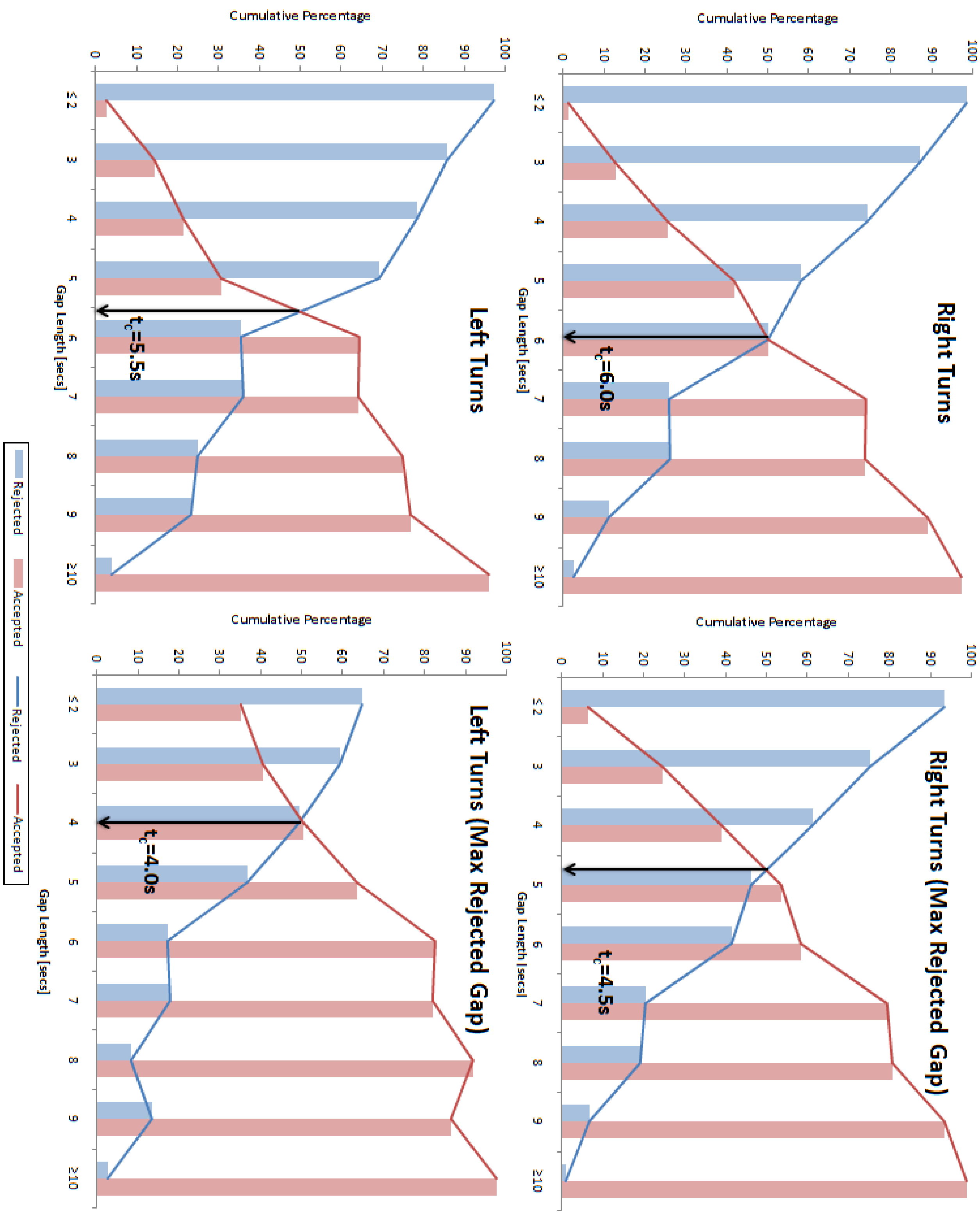

FIGURE 3 Raff Method

The results of the Raff Method are similar to those of the other methods. By using the maximum rejected gap variation the passive driver bias was eliminated thereby lowering the critical gap values. This method was both easy to implement and utilized all of the data available. This method has the added benefits of being easy to display graphically and easy to explain to those unfamiliar with gap acceptance theory. Describing the critical gap as the gap length corresponding to the 50-50 accept or reject decision point is easy to justify logically.

Cumulative Acceptance Method

The Cumulative Acceptance Method is the method described in the commonly used text entitled *Introduction to Traffic Engineering: A Manual for Data Collection and Analysis* by Thomas R. Currin (8). As this is an important resource for practitioners it was a method that warranted inclusion in this research effort.

Implementation

The underlying principle of this method is to identify a gap that would be acceptable to 85 percent of drivers. To do this the count of accepted gaps are binned by gap length. Gap length bins of 0.25 seconds were used as described in the aforementioned manual. Next, for each gap length, the cumulative percentage of accepted gaps is tabulated. According to this method, the critical gap is defined as the gap length where the cumulative percentage is greater than or equal to 15 percent. Note that the cumulative percent accepted first exceeds 15 percent at a gap length of 7.25 seconds, so this is the critical gap as determined by this method.

Sample Size Requirements

Since this method only uses accepted gaps and not rejected gaps, a larger data set is required for reasonable conclusions to be drawn. The usable data from a sample further reduces when gaps over 12 seconds are excluded, necessitating a large sample size for meaningful results.

Results

The Cumulative Acceptance Method was employed to analyze the data from the field study. **Figure 4** presents the results for right and left turning maneuvers for both standard analysis and with the maximum gaps less than 12 second variation.

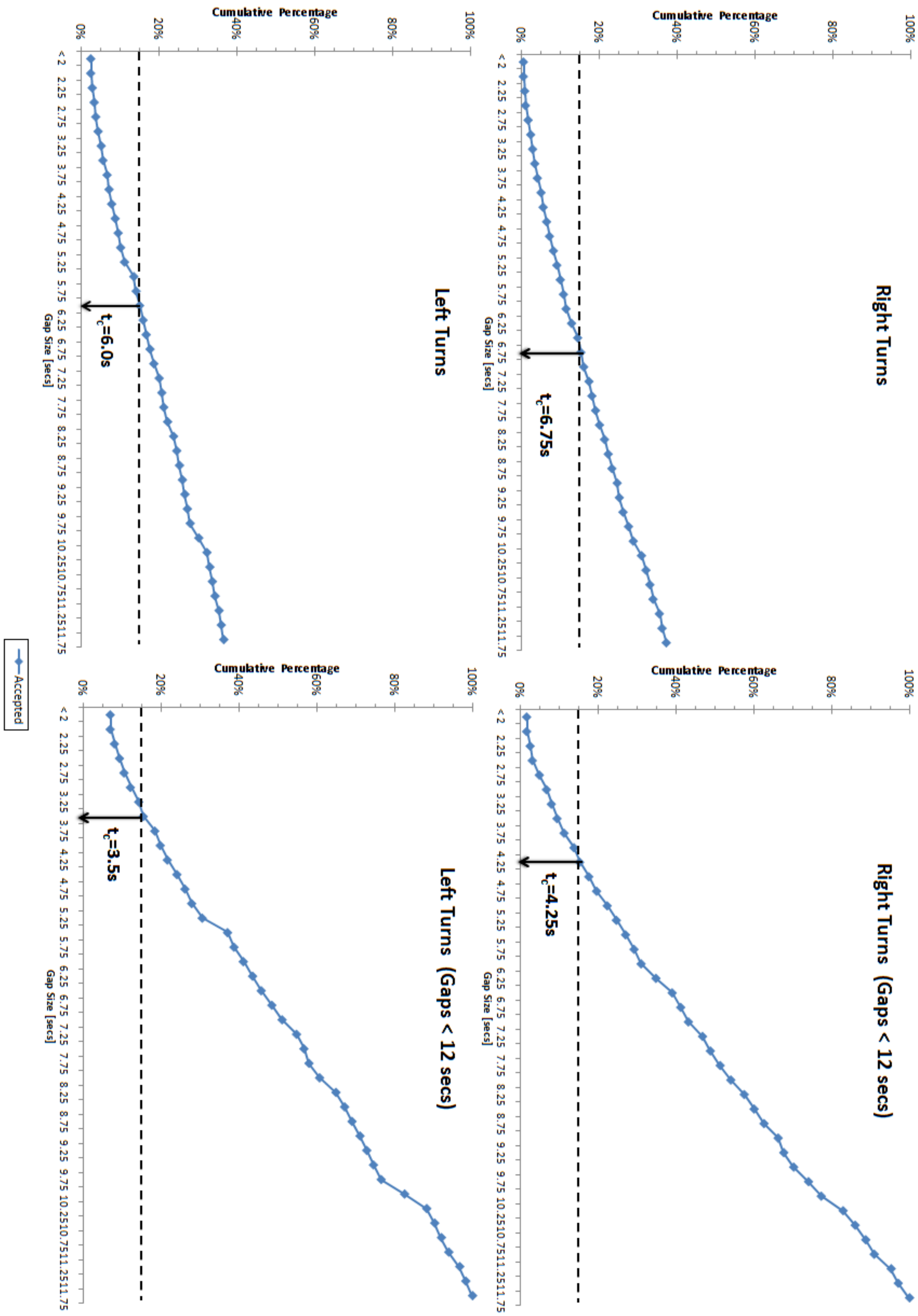

FIGURE 4 Cumulative Acceptance Method

The variation of excluding gaps less than 12 seconds clearly makes a profound difference with this method. The cumulative percentage of accepted gap curves without the variation only approaches 40 percent at 12 seconds as many of the recorded accepted gaps were greater than 12 seconds. This results in a much higher critical gap than with the variation. This variation is not included in the aforementioned manual, meaning that sites with a high proportion of large gaps will show skewed results if the methods outlined in the manual are followed.

Overall, this method gives results similar to those of other methods and is quite simple to implement. The drawback of this method is that the rejected gap data is not utilized meaning a large sample size is need for meaningful results.

Equilibrium of Probabilities

This method has a strong correlation to the fundamental reasoning behind the likelihood maximization logic used in the Troutbeck Method. The variation where only the maximum rejected gaps, not all rejected gaps, are used is almost identical to the Troutbeck Method but without the iterative calculations.

Implementation

The implementation of this strategy follows that proposed by Ning Wu in his paper published in 2006 (9). His tabular calculation of acceptance probabilities mirrors those used by Troutbeck without the iterative calculations. Using a spreadsheet based tabulation; the resulting critical gap value is very close to the thought arrived at by the more computationally intensive Troutbeck Method (9). This is particularly true with the maximum accepted gap variation which more closely mirrors the Troutbeck variation (9). To employ this method, all gaps, both accepted and rejected, are ordered by gap length. Based on whether each of these gaps was rejected or accepted, a model of the maximum likelihood of a gap acceptance decision for gap lengths is developed. This model is able to estimate the critical gap for the sample of gap data analyzed.

Sample Size Requirements

Since this method utilizes both accepted gap and rejected gap data, a smaller sample size relative to other methods is necessitated to obtain meaningful results. All driver choices are reflected in this method of analysis.

With the maximum rejected gap variation, some of the collected data is not used, so a larger sample size is required for meaningful results.

Results

The Equilibrium of Probabilities Method was employed to analyze the data from the field study. **Figure 5** presents the results for left and right turning maneuvers.

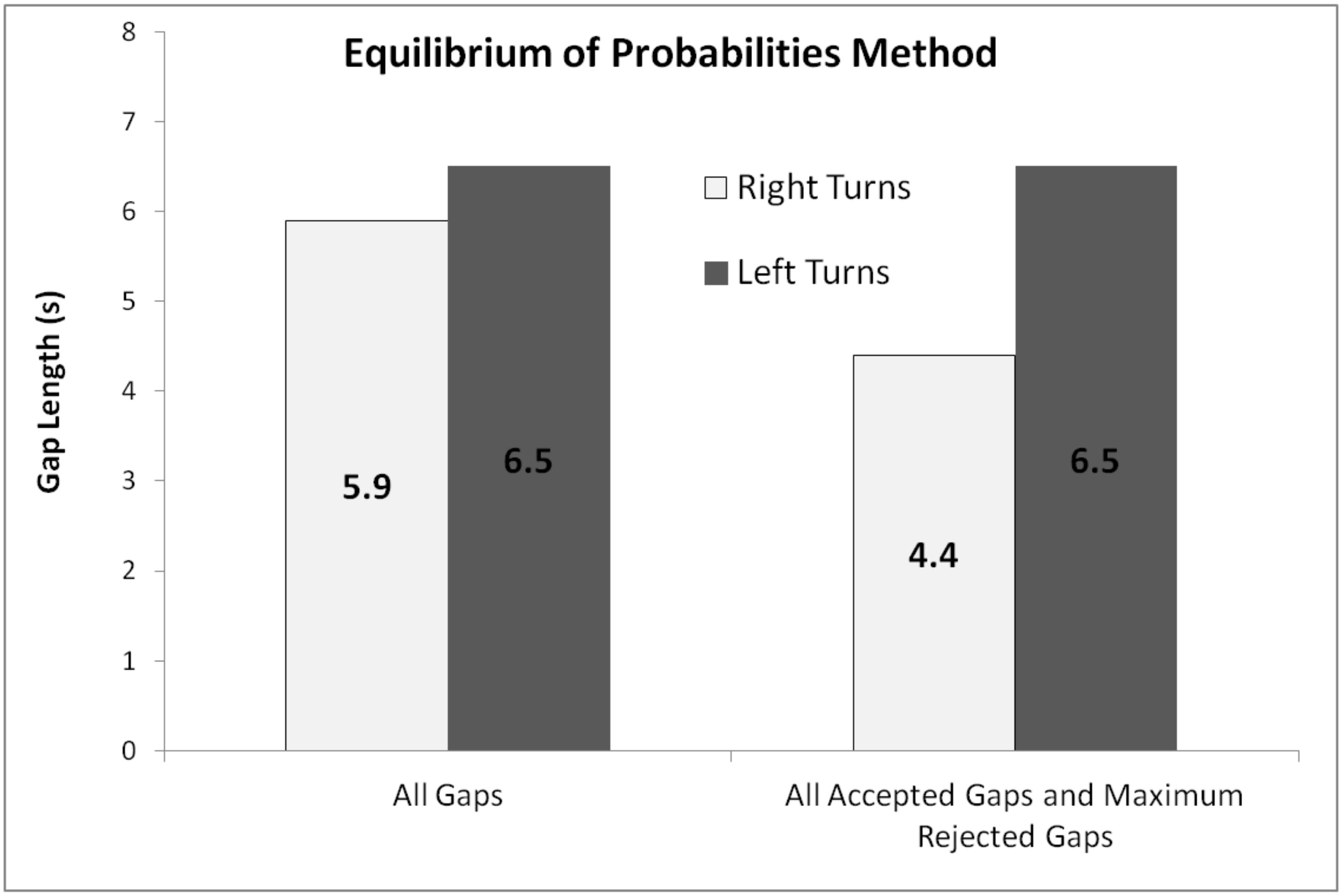

FIGURE 5 Results of Equilibrium of Probability Method Analysis

The results are similar to those from other methods of estimating critical gap. The maximum gap rejected variation showed mixed effects; lowering the right turn critical gap, but not showing any effect on the left turn critical gap.

Overall, this method was fairly simple computationally, although far more time consuming than some of the other methods previously described. Using both the accepted and rejected gap data, this method makes good use of all data on driver behavior collected in the field. Being a relatively new method, it has not been widely used to this point, but given its computational advantages over the Troutbeck Method, it may become more prevalent.

Fit Maximization Method

This method has been around a long time in principle, but the implementation as described below is new to this research initiative. The principle goes back to critical gap as described by D. R. Drew in his traffic flow theory book from the late 1960's (10). His suggestion was that critical gap should be defined as the gap length such that an equal percentage of the population would accept a large gap and reject a smaller gap. Under the assumption that the study sample is representative of the entire population, this would correlate to an equal number of gaps smaller than the critical gap being rejected and larger than the critical gap being accepted. For this research initiative, this statement was modified slightly to find the critical gap that would result in the most gaps larger than the critical gap being accepted and smaller than the critical gap being rejected. This is a bit of a departure from Drew's definition, but the resulting critical gap would be the one that maximizes the number of gaps that fit into the correct position (ie. smaller gaps rejected and larger gaps accepted).

Implementation

The implementation of this method utilized a spreadsheet based algorithm that, for any guess at critical gap, returned the number of gaps that would have been fit that critical gap guess. By trying a variety of critical gaps, the one that maximized the logical gap fits could be pick. An example of such a spreadsheet is presented in **Figure 6**.

tc	4	4.25	4.5	4.75	5	5.25	5.5	5.75	6	6.25	6.5	6.75	7	7.25	7.5	7.75	8
# < Rej	1279	1307	1336	1355	1378	1391	1408	1424	1440	1451	1464	1470	1478	1481	1487	1495	1499
# > Acc	1412	1398	1390	1375	1366	1350	1338	1325	1313	1303	1282	1258	1245	1235	1215	1204	1191
Sum	2691	2705	2726	2730	2744	2741	2746	2749	2753	2754	2746	2728	2723	2716	2702	2699	2690

FIGURE 6 Example of Fit Maximization Reduced Data

A variation where only the maximum rejected gaps, not all rejected gaps was also considered. This variation is more closely related to Drew's definition of critical gap.

Sample Size Requirements

Since this method utilizes both accepted gap and rejected gap data, a smaller sample size relative to other methods is necessitated to obtain meaningful results. All driver choices are reflected in this method of analysis.

With the maximum rejected gap variation some of the collected data is not used, so a larger sample size is required for meaningful results.

Results

The Fit Maximization Method was employed to analyze the data from the field study. **Figure 7** presents the results for left and right turning maneuvers.

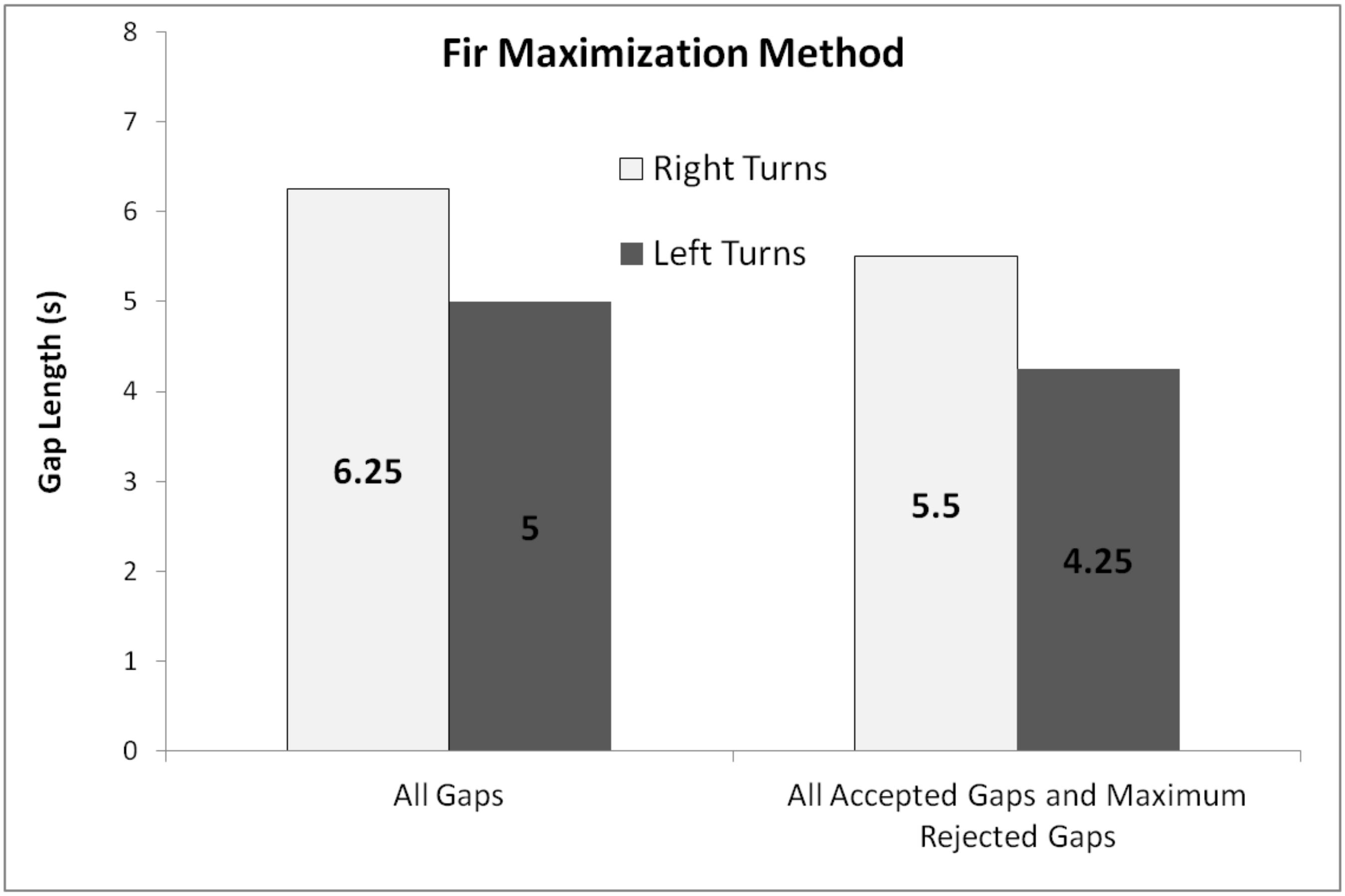

FIGURE 7 Results of Fit Maximization Method Analysis

The results are similar to those of other methods of estimating critical gap. The maximum gap rejected variation slightly reduced both the right turn and left turn critical gap estimates.

Overall, this method was computationally simple and based in sound logic. Using both the accepted and rejected gap data this method makes good use of the all data on driver behavior collected in the field. As this method, at least in this form, has never been used beyond the scope of this research initiative it should be tested under other, varied conditions to test its performance.

Comparison of Results by Method

The five methods, ten including variations, all had their relative merits. All methods except for the Average Accepted Gap Method resulted in estimates of critical gap. The Average Accepted Gap, Cumulative Acceptance, and Raff Methods were the most computationally simple followed closely by the Fit Maximization Method. Of the methods compared, the Equilibrium of Probabilities Method was the most computationally demanding. The Raff, Equilibrium of Probabilities, and Fit Maximization Methods utilized both the accepted and rejected gap data, requiring a smaller sample size. The Average Accepted Gap and Cumulative Acceptance Methods used only accepted gap data requiring a larger sample size for meaningful results. The variation of excluding gaps over 12 seconds seemed to make so of the resulting critical gap values more in line with expectations, but causes the loss of some of the data collected. Similarly, the maximum rejected gap variation seems to result in values that more accurately reflect the driver population, but causes the loss of some of the data collected. The relative merits of each of the method are presented in **Table 2**.

TABLE 2 Merits of Analysis Methods

Methods	Variation	Estimates Critical Gap	Ease of Use	Resulting Sample Size
Average Accepted Gap	All accepted gaps Accepted gaps < 12 seconds	No	Very Easy	Poor Very Poor
Raff Method	All gaps All accepted gaps and maximum rejected gaps	Yes	Very Easy	Very Good Good
Cumulative Acceptance	All accepted gaps Accepted gaps < 12 seconds	Yes	Very Easy	Poor Very Poor
Equilibrium of Probabilities	All gaps All accepted gaps and maximum rejected gaps	Yes	Difficult	Very Good Good
Fit Maximization	All gaps All accepted gaps and maximum rejected gaps	Yes	Easy	Very Good Good

To see whether or not different analysis methods lead to different results, estimated critical gaps were compared across methods. For completeness, the average accepted gap as determined using the Average Accepted Gap Method was included as it is sometimes used as a proxy for critical gap. The values are presented in **Figure 8** along with a comparison to HCM values as discussed in the next section.

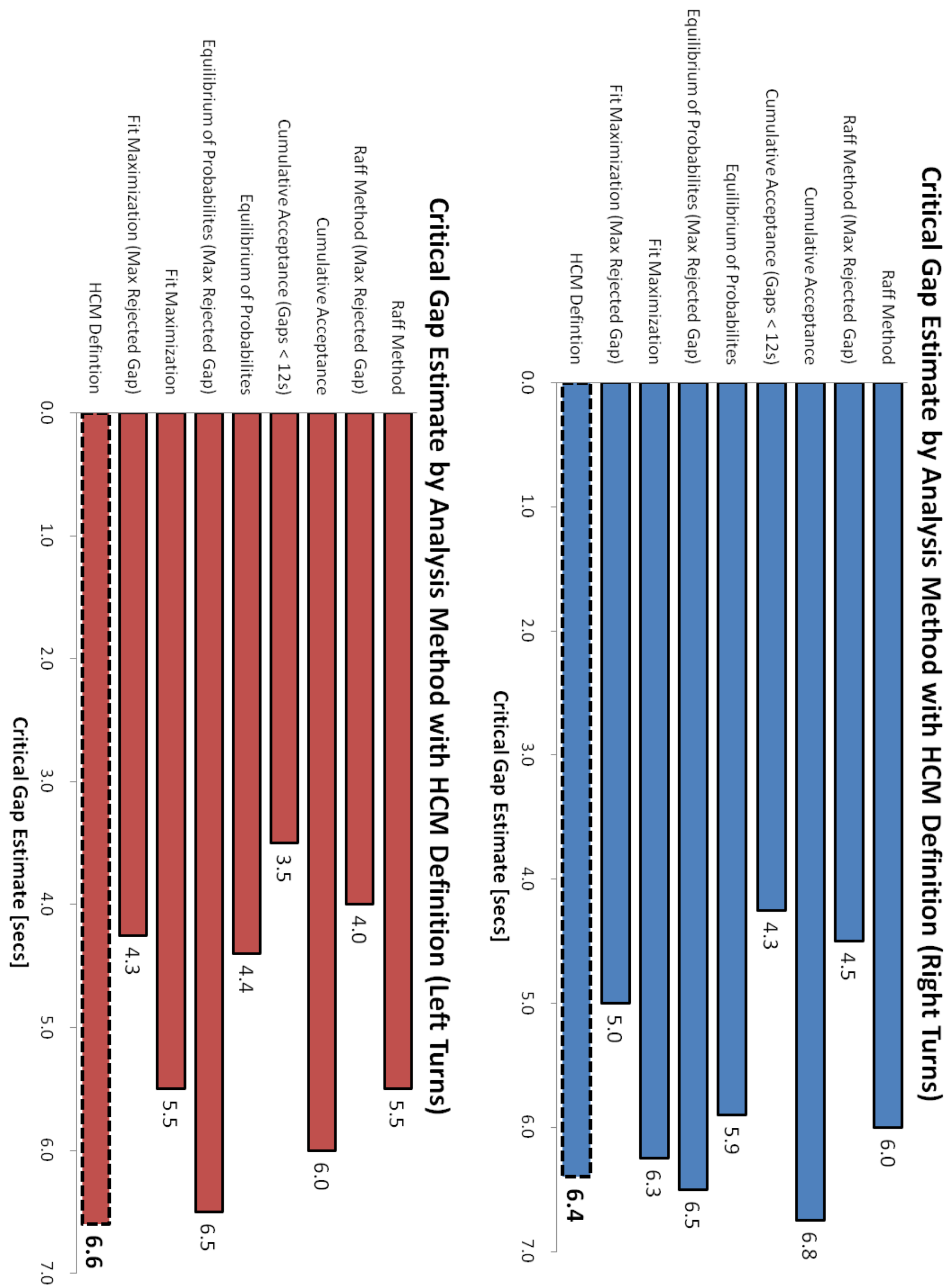

FIGURE 8 Comparison of Critical Gap by Analysis Method

As the figure shows, there is a good deal of variation in the results of the analysis methods compared. The right turn critical gap estimate varied from 4.25 seconds to 6.75 seconds, and the left turn critical estimate varied from 3.5 seconds to 6.5 seconds. As the critical gap estimate depends of the definition of critical gap, there is no way to tell which values are "most correct," however general consensus between methods is a good indicator of a reasonable value. Additionally, the values are relatively close to values published in other literature.

One's intuition would suggest that making a left turn would require a larger gap than a right turn since an extra lane(s) needs to be crossed. However, empirical field data suggested that this was not the case. The larger critical gap values for right turns may be explained by the fact that left turns are harder to complete and thus drivers must make a riskier maneuver. Whereas with right turns, there are more opportunities to make the turning movement so drivers will wait for a safer gap.

HCM Comparison

One way of determine the validity of the results of the analysis methods is to compare them to the standard values reported in the *Highway Capacity Manual 2000*. Such a comparison is presented in **Figure 8**.

However, it should be understood that the HCM definition value may not be applicable to all of the locations and conditions under which the study was conducted. The conditions that had the greatest impact were the intersection geometry which was a T-intersection for all locations and the number of lanes on the major street which was taken to be the weighted average between the actions recorded at two and four lane roadways. The HCM definition should therefore not be considered the "true value" but rather a value of critical gap worthy of comparison. For many methods, the critical gap estimates are quite close to the HCM value of critical gap. Overall, the method that most closely compared to the HCM definition was the Equilibrium of Probabilities method with the maximum rejected gap variation.

CONCLUSIONS

Given the significant role of gap acceptance data across a myriad of widely used traffic analyses, there is an inherent need to better understand the direct impacts associated with which gap acceptance methodology is being utilized in a given study. The research presented herein provides a major step forward in understanding the unique differences across gap acceptance methodologies. More specifically, five gap acceptance data analysis methods were identified with two variations of each. All methods except for the Average Accepted Gap Method resulted in estimates of critical gap. The Average Accepted Gap, Cumulative Acceptance, and Raff Methods were the most computationally simple followed closely by the Fit Maximization Method. Of the methods compared, the Equilibrium of Probabilities Method was the most computationally demanding. The Raff, Equilibrium of Probabilities, and Fit Maximization Methods utilized both the accepted and rejected gap data, requiring a smaller sample size to reach statistical significance. The Average Accepted Gap and Cumulative Acceptance Methods used only accepted gap data and required a larger sample size for meaningful results.

The variation of excluding gaps over 12 seconds seemed to make some of the resulting critical gap values fall more in line with expectations, but caused a reduction in sample size. Similarly, the maximum rejected gap variation seems to result in values that more accurately reflect the driver population, but significantly decreases the sample size.

Methods, such as the Siegloch Method, were excluded because their application did not match the study conditions as all observations took place during unsaturated conditions. While there are tools available to help with the implementation of computationally intensive analyses, methods such as the Troutbeck Method were excluded from this study but would be worth exploring in further research.

Arguably the most important finding of this research is that the method used for analysis, at times, resulted in statistically different results. This fact, highlights the need for a more widespread understanding of the results obtained using a selected methodology. Yet another important finding was the direct applicability of several of the methods considered herein and their close approximations of critical gap values as defined by the Highway Capacity Manual.

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